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Building Rothermel fire behaviour fuel models by Genetic Algorithm optimization

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Abstract

A method to build and calibrate custom fuel models was developed by linking Genetic Algorithms (GA) to the Rothermel fire spread model. GA randomly generates solutions of fuel model parameters to form an initial population. Solutions are validated against observations of fire rate of spread via a goodness-of-fit metric. The population is selected for its best members, crossed over, and mutated within a range of model parameters values, until a satisfactory fitness is reached. We showed that GA improved the performance of Rothermel model in three published custom fuel models for litter, grass, and shrub fuels (RMSE decreased by 39%, 19% and 26%). We applied GA to calibrate a mixed grass-shrub fuel model, using fuel and fire behaviour data from fire experiments in dry heathlands of Southern Europe. The new model had significantly lower prediction error against a validation dataset than either standard or custom fuel models built using average values of inventoried fuels, and predictions of the Fuel Characteristics Classification System. GA proved a useful tool to calibrate fuel models and improve Rothermel model predictions. GA allows exploration of a continuous space of fuel parameters, making fuel model calibration computational effective and easily reproducible, and does not require fuel sampling. We suggest GA as a viable method to calibrate custom fuel models in fire modelling systems based on the Rothermel model.

Running head: Optimization of fuel models by Genetic Algorithms

Brief summary

Calibration of Rothermel fuel models by Genetic Algorithms improves simulation of fire behaviour in both published and newly measured fire experiments. Genetic Algorithms are computational effective, make calibration reproducible, and do not require fuel sampling. A fuel model for European heathlands was calibrated using the *gaRoth* ( ) function of the Rothermel package for R.

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Introduction

The Rothermel (1972) fire spread model is one of the most used models to simulate the forward rate of spread (ROS) at the head of a surface fire (Sullivan 2009), and is the primary fire spread model of many fire prediction systems (Lopes *et al.* 2002; Ferragut *et al.* 2008; Sullivan 2009; Finney *et al.* 2011; Andrews 2013).

In the Rothermel model, ROS is simulated as a function of topography, weather and a "fire behaviour fuel model" (hereafter: fuel model) that consists of a number of fuel parameters for a given fuel complex (Albini 1976; Burgan and Rothermel 1984), including fuel load (kg m^{-2}) and surface area-to-volume (SAV) ratio (m^2 m^{-3}) for different fuel categories and size classes; fuel bed depth (cm); dead fuel moisture of extinction (%); heat content (kj kg^{-1}) of dead and live fuels (see Table 1). "Standard" stylized fuel models have been built for most areas in North America (Rothermel 1972; Anderson 1982; Scott and Burgan 2005) to facilitate ROS simulation. However, standard fuel models are often inappropriate for some situations, such as simulating the effects of fuel treatments on potential fire behaviour (Sandberg *et al*. 2007; Cruz and Alexander 2010), and many studies have identified the need to build custom fuel models to represent local conditions (e.g., Davies 2006; Cruz and Fernandes 2008; Cai *et al*. 2014).

Several methods have been used to build custom fuel models (Hough and Albini 1978; Rothermel and Rinehart 1983; Burgan 1987; Cruz and Fernandes 2008; Wu *et al*. 2011). Generally, fuel models are built using inventoried average characteristics of fuelbeds, but often without testing their performance (e.g., Wu *et al*. 2011). A more rigorous approach is that of fuel model calibration (Hough and Albini 1978; Burgan 1987; Cruz and Alexander 2010), which iteratively adjusts fuel model parameters based on their performance against observed ROS or flame length (Rothermel and Rinehart 1983; Burgan and Rothermel 1984; van Wilgen *et al*. 1985; Cai *et al*. 2014). This was usually done by starting from either standard fuel models, or mean or median characteristics of sampled fuelbeds, followed by subjective adjustments until a satisfactory match between predictions and observations is achieved (Hough and Albini 1978; Burgan 1987; Cruz and Alexander 2010). These empirical adjustments usually target first those parameters to which the Rothermel model is more sensitive, such as fuel bed depth (Catchpole *et al*. 1998), and 1-h fine fuel load and SAV (Burgan 1987), and only then dead fuel moisture of extinction, 10-h and 100-h parameters, and heat content (Burgan and Rothermel 1984).

However, empirical calibration suffers from several shortcomings. First, the process is highly subjective and hardly reproducible. Second, due to non-linearities in the relationships between fuel model parameters in Rothermel system of equations, it is difficult to determine how adjustments will affect the ROS output (Burgan 1987; Jolly 2007). For example, an increase in fuel load can increase the reaction intensity but also decrease the wind factor, which are both directly proportional to ROS in Rothermel equation (Rothermel 1972). Therefore, an increase in fuel load can either increase or decrease ROS predictions (Burgan 1987). Third, empirical calibration may underestimate the fire spread because of the difficulty in quantifying the proportion and packing ratio of the fine fuel that is actually carrying the fire (Cruz and Fernandes 2008).

To overcome problems related to the calibration of stylized fuel models, two solutions were adopted. Sandberg *et al.* (2007) proposed a reformulation of the Rothermel model that allows the direct use of mean or median fuel parameter values measured in the field. However, this reformulation has yet to be fully tested (Schaaf *et al*. 2007; Prichard *et al*. 2013). Using the original Rothermel model coupled with mathematical optimization of fuel models is considered more effective (Cruz and Alexander 2010).

In previous studies, mathematical optimization of fuel models was obtained by systematically

perturbing their parameters by discrete increments within a predefined range ("search space"), and choosing the values that maximize the fit between predicted and observed fire behaviour (Hough and Albini 1978; Cruz and Fernandes 2008). However, this approach can result in high computational load and incomplete exploration of the search space.

Genetic Algorithms (Holland 1975) are a method for solving optimization problems which have been successfully used to calibrate multi-dimensional and nonlinear models in several fields of ecology and natural resource management (e.g., Stockwell and Noble 1991; Wang 1991), including fire science (Finney 2004; Lautenberger *et al.* 2006; Wendt *et al.* 2013). They use stochastic search rules and explore a continuous search space, which make them suitable to calibrate fuel model parameters in a computationally effective way.

In order to propose an objective, reproducible, and effective method to build Rothermel fuel model, this paper aims to: (1) test if optimization by Genetic Algorithms (GA) improves the accuracy of custom fuel models for the Rothermel system of equations, (2) calibrate a custom fuel model for dry heathlands dominated by the dwarf shrub *Calluna vulgaris* (L.) and the perennial grass *Molinia arundinacea* Shrank by GA optimization, using fuel, weather, and fire behaviour data measured under experimental conditions to demonstrate GA procedures, and (3) evaluate the performance of the GA-optimized fuel model for dry heathlands against standard fuel models, custom fuel models using mean values from the field, and predictions of the reformulation of Rothermel model by Sandberg *et al.* (2007).

Methods

GA optimization settings

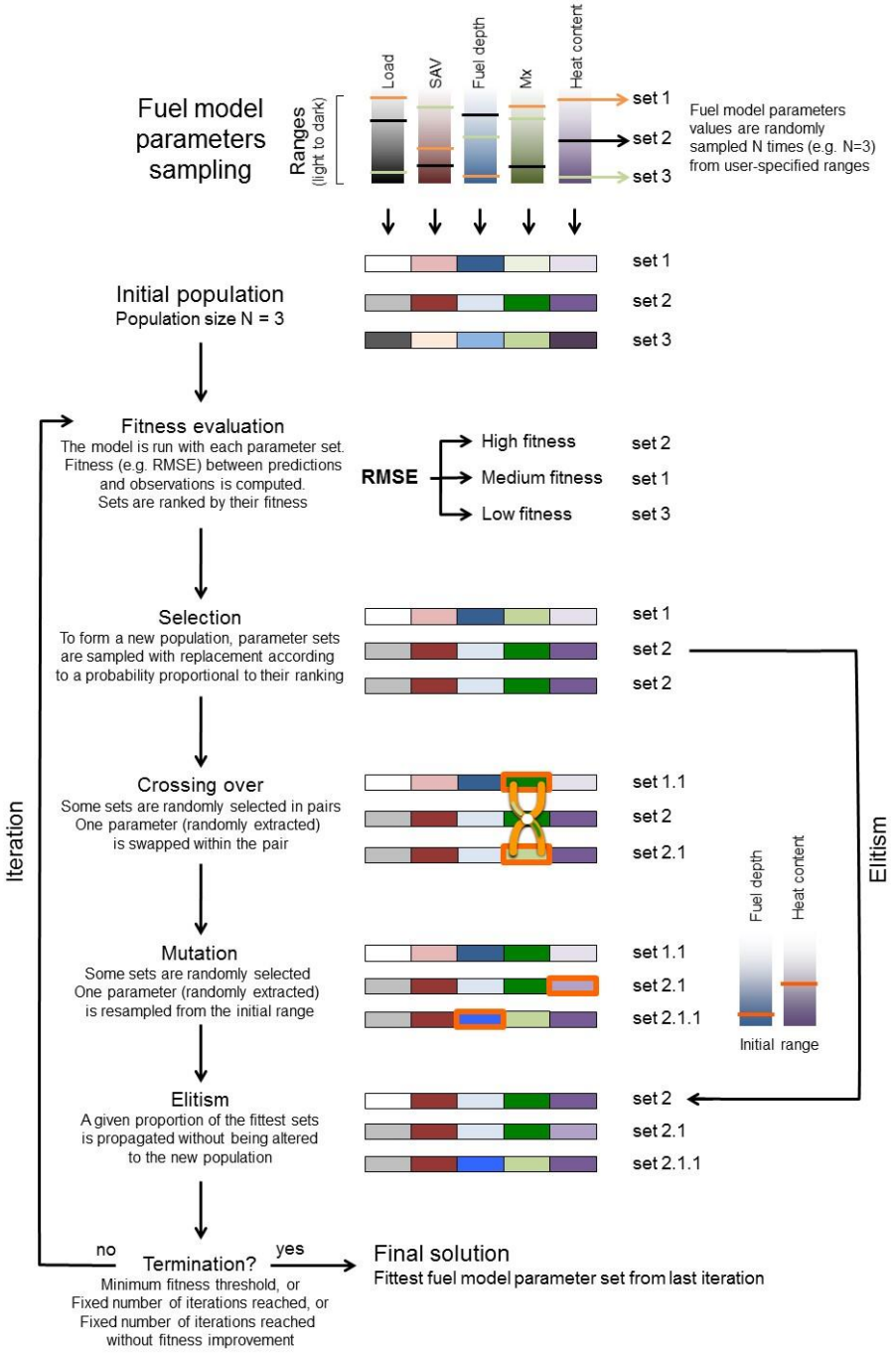
When applying GA to fuel model calibration, the first step is to generate the initial set of fuel model parameters. This is done via random sampling within a user-defined range for each of the parameters (Fig. 1). More than one set of parameters are generated at the same time to explore the largest part of the search space. Then, Rothermel model is run using all sets of parameters, and the fitness of each (i.e., the agreement between predicted and observed ROS) is evaluated. Several iterations are carried out, where the fittest parameter sets from one iteration are retained for inclusion in the next one ("selection"), recombined ("crossing over"), and randomly perturbed ("mutation") (Fig. 1). A successful variant of the process is to allow some of the best parameter sets to carry over unaltered through selection ("elitism"). The process is iterated until a maximum number of iterations has been produced, or a given fitness has been reached (Goldberg 1989).

We ran GA calibration by using the GA package (Scrucca 2013) for the R statistical framework, version 3.0 (R Core Team 2013). This package allows the users to customize GA rules for selection, crossing over, mutation and elitism. Several studies evidenced how GA performs differently with different problems, so rules for one GA does not generalise to all cases (Eiben *et al.* 1999). Much research has been devoted to find rules that avoid premature convergence to non-optimal solutions (Grefenstette 1986). As a first application of GA to fuel model calibration, we decided to adopt default settings of the GA package (Scrucca 2013): population size =50, probability of crossing over =80%, probability of mutation =10%, elitism =5%, selection operator = fitness proportional, crossover operator = local arithmetic, mutation operator = uniform random value replacement. The maximum number of iterations was set to 9999.

The fitness function to be minimized was Root Mean Square Error (RMSE) between predicted and observed ROS. RMSE has already been used to optimize fuel models (Cruz and Fernandes 2008). Since it is expressed in units homogenous to the dependent variable, under the assumption of homoscedasticity, it is directly proportional to values of the modeled variable, which makes it apt to correct for the common underprediction bias in the Rothermel model (Catchpole *et al.* 1993; Cruz

and Alexander 2013). To simulate ROS, we used the system of equations implemented in the *ros* ( ) function of the Rothermel package for R (Vacchiano and Ascoli 2014). This function uses the imperial version of the Rothermel model (Rothermel 1972), includes corrections to the original model by Frandsen (1973) and Albini (1976), the dynamic fuel load transfer function (Scott and Burgan 2005), and has removed the maximum wind factor limit (Andrews *et al.* 2013).

Fig. 1. Flowchart for the optimization of fire behaviour fuel model parameters by Genetic Algorithms (GA). As example, we used a population size = 3 and a fuel model of five parameters: load, surface area-to-volume ratio (SAV), fuel depth, moisture of extinction (Mx), heat content. GA rules for selection, crossing-over, mutation, and elitism are described in the Methods section.



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GA optimization testing

First, we searched the wildland fire science literature for studies testing the Rothermel model (reviewed by Cruz and Alexander 2013), and reporting the following information: (i) a dataset of observed ROS; (ii) measures of fuel moisture, wind speed and slope steepness associated to each ROS observation; (iii) a custom fuel model calibrated using published ROS observations; (iv) a dataset of inventory or laboratory fuel characteristics, from which to infer ranges of fuel model parameters to force GA optimization. Following these criteria, we selected test-run studies for litter (Grabner *et al.* 1997; 2001), grass (Sneeuwjagt 1974; Sneeuwjagt and Frandsen 1977) and shrub fuels (van Wilgen 1984; van Wilgen *et al.* 1985), with the following modifications:

(a) for the litter study, as several fires in Grabner *et al.* (1997) did not present alignment of wind and slope vectors, we retained only ROS observations with a difference between aspect and wind direction <30°, indicative of better alignment of vectors;

(b) for the grass study, we used a unique fuel model as calibrated in Sneeuwjagt (1974) using ROS observations reported by Sneeuwjagt and Frandsen (1977) – who, by contrast, averaged fuel input parameters measured at each fire site;

(c) for the shrub study, to be consistent with van Wilgen *et al.* (1985) we considered the fuel model as "static", despite the presence of live herb fuels would require a "dynamic" model.

A total of 10, 42 and 14 ROS observations were used, together with associated wind speed, slope and fuel moisture, for the litter, grass and shrub study, respectively. We constrained GA optimization of fuel model parameters using ranges of fuel characteristics specified in each study (Table 1). For each test study we ran one GA optimization. Then, we computed the goodness-of-fit of observed vs. predicted ROS using either custom fuel models from the test studies, or GA optimization. Goodness-of-fit metrics for Rothermel predictions were: root mean square error (RMSE), mean absolute error (MAE), mean absolute percent error (MAPE), and mean bias error (MBE) (Cruz and Alexander 2013). Finally, we did a t-test to identify significant differences ($p<0.05$) between means of residuals of published and GA-optimized models under the hypothesis that the latter would produce a lower prediction error.

Heathland fuel model data

We used GA optimization to calibrate a mixed grass-shrub fuel model (GA-heath) for dry heathlands of Southern Europe, which are lowland heaths on mineral soil dominated by *Calluna* and *Molinia* (Lonati *et al.* 2009). We selected this vegetation for the following reasons:

(i) heathlands satisfy Rothermel model assumptions, i.e., fuel is homogenous, well mixed, continuous, contiguous to the ground, vertically oriented (Rothermel 1972);

(ii) the flammability of heath fuels is well documented (e.g., Davies and Legg 2011; Santana and Marrs 2014);

(iii) management of European heathlands by prescribed burning (Davies *et al.* 2009; Fernandes *et al.* 2013) could benefit from a custom fuel model targeted to dry heaths.

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Table 1. Ranges of fuel model parameters used to constrain GA optimization in test-run fuelbeds (litter, grass and shrub fuels) and for the heathland fuel model (GA-heath) calibration. Single values indicate that no range was applied. Dashes indicate absence of the parameter from the fuel model.

Variables	**Litter fuel[A]**	**Grass fuel[B]**	**Shrub fuel[C]**	**GA-heath[D]**
Fuel model parameters				
Model Type	Static	Static	Static	Static
Load 1-h (t ha^{-1})	5.24 – 8.65	0.5 – 4.9	1.56 – 6.24	1.50 – 7.24
Load 10-h (t ha^{-1})	0.36 – 1.51	–	0.4 – 1.2	–
Load 100-h (t ha^{-1})	0.57 – 6.49	–	0.06 – 0.18	–
Load Live Herb (t ha^{-1})	–	–	1 – 6	–
Load Live Woody (t ha^{-1})	–	–	0.64 – 6.72	2.90 – 10.40
SAV 1-h (m^2 m^{-3})	4921–11423	4600 – 14800	4200 – 8000	6640 – 10036
SAV 10-h (m^2 m^{-3})	358	–	358	–
SAV 100-h (m^2 m^{-3})	98	–	98	–
SAV Live Herb (m^2 m^{-3})	–	–	4200 – 6500	–
SAV Live Woody (m^2 m^{-3})	–	–	4200 – 5500	8810 – 10560
δ (cm)	6–76	9 – 53	100 – 200	19 – 70
M_x (%)	12–40	12 – 25	20 – 40	25 – 50
Heat content 1-h (kJ kg^{-1})	18622	18622	18000 – 22000	18719 – 19919
Heat content 10-h (kJ kg^{-1})	18622	–	18000 – 22000	–
Heat content 100-h (kJ kg^{-1})	18622	–	18000 – 22000	–
Heat content Live Woody (kJ kg^{-1})	–	18622	18000 – 22000	20000 – 22504
Fire variables				
Rate of spread (m min^{-1})	0.3 – 10.1	0.2 – 61.0	2.4 – 53.4	0.9 – 26.3
Wind speed (km h^{-1})	0 – 8	3.1 – 11.3	3.7 – 12.8	0.4 – 7.9
Dead fuel moisture (%)	4 – 20	8 – 15	2 – 13	10 – 27
Live fuel moisture (%)	–	55 – 170	91 – 147	50 – 70
Slope (%)	1 – 24	0 – 7	0	0

Fuel model parameters: 1-h: dead fuels below 6 mm in diameter; 10-h: dead fuels between 6 and 25 mm in diameter; 100-h: dead fuels between 25 and 75 mm in diameter; SAV: surface area to volume ratio; δ: fuel bed depth; M_x: dead fuel moisture of extinction; Heat: high heat content of combustion.

[A] Fuel load after Table 6 in Grabner *et al*. (2001); SAV, δ, M_x and Heat content after standard fuel models tested in Grabner *et al*. (2001).

[B] Fuel load, SAV, δ, and Heat content after Sneeuwjagt and Frandsen (1977); M_x after group "Grass" of standard fuel models (Scott and Burgan 2005).

[C] Fuel load after Table 1 in van Wilgen *et al*. (1985); SAV and Heat content after Table 2 and Table 3 in van Wilgen (1984); δ after van Wilgen *et al*. (1985); M_x after group "Shrub" of standard fuel models (Scott and Burgan 2005).

[D] Fuel load and δ from field measures; SAV, M_x and Heat content after Fernandes and Rego (1998), Spielmann (2009), Davies and Legg (2011), Santana and Marrs (2014).

The GA-heath model was optimized against ROS observations recorded in nine wind-driven field fire experiments in Northwest Italy (Ascoli *et al*. 2013; Vacchiano *et al*. 2014). Fire experiments were carried out on a flat terrain in *Calluna* stands in the "building" phase, i.e., a growth stage 8 to 15 years old, characterized by a dense uniform canopy and many slender stems (Davies *et al*. 2009). Eight fire experiments were conducted under moderate to dry weather on four different burn days during the winter dry season (two fires per day) when grass fuel is fully cured. Two burn days were in the early dry season (January-February), during the legal prescribed burning season, and others two burn days were in the late dry season (March), when wildfires usually occur in heathlands of NW Italy (Ascoli and Bovio 2010). The ninth fire experiment was conducted at a fifth burn day under marginal burning conditions (i.e., high fuel moisture and low wind speed). To let the fire front reach a pseudo-steady state, each fire experiment was ignited upwind by line ignitions ranging from 25 to 50 m in length, and the fire was allowed to spread for 50 to 80 m before being suppressed along a fuel break.

Fine fuel (<6 mm in diameter) moisture was assessed at each fire experiment by collecting five samples of dead and live *Calluna* crowns and dead *Molinia* leaves. Fresh samples were weighed in the field using a portable scale, and then oven-dried in laboratory at 90°C to constant weight. Fuel moisture was computed on a dry weight basis.

During each fire experiment, the spread rate was assessed at a microplot scale (Fernandes *et al.* 2001) by measuring the arrival time of the fire front at the vertices of a triangle and computing ROS according to the trigonometric method developed by Simard *et al.* (1984). At each fire experiment site (hereafter: fire site), 10 equilateral triangles (10 m side) were visualized using 2 m rods placed at each triangle vertex. During fire progression, six observers recorded the time of arrival of the fire front at each rod. Wind speed and direction were assessed every 30 seconds by two anemometers positioned at a height of 2 m above ground, upwind to the experimental plot. In a few cases, marked changes in wind direction occurred during the burn. However, the microplot approach allowed data recorded during backfire and flank fire phases to be discarded. Acceleration phases (<10 m from the ignition) were also excluded. A total of 40 ROS observations were retained.

We used data from both a fuel inventory and the literature to populate the initial ranges of fuel parameters for the GA-heath model (Table 1). Fuel characteristics were measured on all nine fire sites, plus three additional *Calluna* stands in the building phase. In total, twelve heath stands were assessed. At each stand, fuels were harvested in six 1 m^2 quadrats and oven-dried in the laboratory at 90°C to determine the load range of 1-h (dead *Calluna* and fully cured *Molinia*) and live woody (*Calluna* live foliage with a diameter <6 mm). Large dead fuels (10-h and 100-h parameters) were not present, while live stems with a diameter >6 mm were not considered, since they largely remain unburned (Davies *et al.* 2009). Presence-absence and depth of shrub and grass fuels were measured every 0.5 m along six linear transects (length =10 m) at each stand. Shrub and grass cover were then computed as 1/20 times the respective presence counts along the transect. Ranges of flammability parameters (SAV, moisture of extinction, heat content) for heath fuels were derived from published studies (Fernandes and Rego 1998; Spielmann 2009; Davies and Legg 2011; Santana and Marrs 2014). The fuel model was conceived as static, consequently the load of cured grass was added to the 1-h class whose SAV and moisture were weighted using the method described in Burgan and Rothermel (1984, Appendix E).

Calibration and validation of the heathland fuel model

ROS observations were divided into a calibration and a validation set. The calibration set included the fire experiment with the highest ROS recorded on each burn day, and the ninth fire experiment burnt under marginal conditions. Following these criteria, 20 ROS observations were assigned to the calibration set and 20 to the validation set, producing a balanced design. We carried out optimization of GA-heath by one Genetic Algorithm run using ROS and environmental conditions from the

calibration set, and fuel parameters ranges reported in Table 1.

Using the validation set and the *ros* ( ) function of the Rothermel package for R (Vacchiano and Ascoli 2014), we computed goodness-of-fit metrics (RMSE; MAE; MAPE; MBE) for model predictions obtained using the following alternative fuel models:

- GA-heath: the heathland fuel model calibrated using GA optimization;
- Stand-GS: the standard fuel model selected among the grass-shrub group (Scott and Burgan 2005) which produced Rothermel predictions with the least RMSE against observations, as determined by the function *bestFM* ( ) from the Rothermel package for R (Vacchiano and Ascoli 2014). Since the grass-shrub group is described by dynamic fuel models, we activated the fuel load transfer function (Scott and Burgan 2005) implemented in the R package, and set live herb fuel moisture at 30%, i.e., complete transfer of fully cured herbaceous fuels to the 1-h class;
- Cust-1: a custom fuel model for *Calluna* heaths, parameterized by averaging fuel characteristics sampled in all 12 heath stands (72 observations);
- Cust-2: a site-specific custom fuel model parameterized by averaging fuel loadings and structure measured at each fire site (six observations per site).

We also computed the fitness metrics for ROS predicted by the Rothermel model reformulation that is implemented in the Fuel Characteristics Classification System (FCCS), version 2.2 (Sandberg *et al.* 2007; Prichard *et al*. 2013). We instructed FCCS to calculate fuel model parameters by selecting *Calluna* in the shrub layer, and *Andropogon gerardii* Vitman in the non-woody layer (grass fuels in FCCS), i.e., a species with a SAV of 4920 m^2 m^{-3} which is adequate for *Molinia* (source: FCCS Inferred variables)[1]. Shrub loads in FCCS are a function of shrub cover and height, therefore we entered median fuel load of *Calluna* (both for all stands pooled – FCCS-1, and for each fire site – FCCS-2) by adjusting the cover of the shrub layer in FCCS until the desired load at the given median depth was reached. Median values of grass fuel loads, fuel bed height and cover (both for all stands pooled, and for each fire site) were entered directly in the non-woody layer (Table 2). Custom fuel moisture scenarios recorded at each fire experiment were used, and the non-woody moisture was set at 30%.

Finally, we assessed the accuracy of each fuel model, including FCCS, by running an analysis of variance (ANOVA) of model residuals and a bootstrapped regression-based equivalence test (Robinson *et al.* 2005). After successfully testing for ANOVA assumptions, we used a least-square difference (LSD) post-hoc test to identify significant differences ($p<0.05$) between means of model residuals. The equivalence tests were carried out on the validation set of experiments by using the function *equiv.boot* ( ) of the package "equivalence" version 0.5.6 (Robinson 2013) for the statistical software R. The amplitude of the equivalence intervals for the intercept and slope of the predicted:observed regression line was set to ±25%. If the two one-sided confidence intervals for the intercept and slope was within the equivalence interval, we could reject the null hypothesis of dissimilarity against a zero intercept and a 1:1 slope, therefore ascertaining the absence of bias and trends in model predictions.

[1]http://www.fs.fed.us/pnw/fera/fccs/inferred_variables/table2_metric/table2_metric.htm

Results

GA optimization testing

GA optimization produced fuel models for the litter, grass and shrub test studies which markedly differed from those published (Table 2). GA-optimized fuel models displayed a lower bulk density, i.e., 23%, 73% and 18% lower than the published value for the litter, grass and shrub models, respectively. The relative packing ratio (ratio of actual to optimum packing ratio for combustion efficiency) (Rothermel 1972) was also reduced by GA optimization (Table 2). However, while the relative packing ratio in the litter model declined from 2.43 to 1.12 (i.e., closer to the optimum) in both the grass and shrub models it diverged from optimum, showing a decrease from 0.37 to 0.12, and from 0.23 to 0.19, respectively.

All goodness-of-fit statistics improved after GA optimization. RMSE decreased by 39%, 19% and 26% for the litter, grass and shrub models, respectively (Table 3). Other statistics also showed a substantial contribution by GA optimization in improving performance relative to the published fuel models. The t-test evidenced that GA optimization significantly reduced the mean of residuals for both the litter and grass fuel model, but not for the shrub one (Table 3).

The heathland fuel model

Fire experiments in heath fuels were carried out under a relatively broad range of weather and fuel moisture conditions: days since last rain, air temperature and humidity, and wind speed ranged between 3 to 24 days, 90 to 25%, 2 to 20°C, and 0.4 to 7.9 km h^{-1}, respectively; fuel moisture ranges were 10-27% for dead fuels (dead *Calluna* and cured *Molinia*), and 50-70% for live fuels (*Calluna* foliage). Notably, in most experiments live *Calluna* had a moisture <60%, i.e., a value close to the lower edge of physiological activity, indicative of winter desiccation, and thus of high flammability (Davies *et al*. 2010). Therefore, even though experiments were carried out under prescribed burning conditions due to safety reasons (Ascoli and Bovio 2013), experiments from the late dry winter season approached wildfire conditions under moderate fire weather. Consequently, we observed a relatively broad array of fire behaviour with ROS ranging from 0.9 to 26.3 m min^{-1}. The complete dataset of ROS observations and environmental conditions during fire experiments is available on Comprehensive R Archive Network[2] as example data (*firexp*) in the Rothermel Package for R (Vacchiano and Ascoli 2014).

GA-heath optimization against the calibration data resulted in a RMSE of 1.67 m min^{-1} and a MAPE of 20%. Three quarters of observed ROS values were predicted with less than ±25% error. Half of these values displayed underprediction, as showed by the predicted vs. observed scatter plot of Fig. 2. Calibrated parameters for the GA-heath model (Table 2) behaved as follows: for 1-h fuels, loadings were within the 50th percentile of inventoried values, and SAV and heat content were close to intermediate values within the range found by experimental studies (Davies and Legg 2011; Santana and Marrs 2014). Conversely, live woody fuels displayed a load higher than the 75th percentile of inventoried values, and SAV and heat content close to the range maximum. Dead fuel moisture of extinction assumed intermediate values of the experimental range (Davies and Legg 2011; Santana and Marrs 2014). Fuel bed depth showed values close to the 95th percentile of the observed range. The fuel bed bulk density was 1.96 kg m^3 and the relative packing ratio was 0.82.

The GA-heath model tested against the validation dataset displayed a MAPE equal to 32%. RMSE, MAE and MBE were 1.81, 1.42 and 0.49 m min^{-1}, respectively. Two thirds of observed ROS values in the calibration dataset were within ±25% error of the predicted values, and 50% of these displayed underprediction (Fig. 2).

[2] http://cran.r-project.org

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Table 2. Parameters of the litter, grass, shrub, and heathland fuel models as determined by GA optimization (GA-litter; GA-grass; GA-shrub; GA-heath), published studies (Pub-litter; Pub-grass; Pub-shrub; Stand-GS3), and inventoried fuels (Cust-1; FCCS-1).

Fuel model	Type	Dead fuel load ($t\ ha^{-1}$)			Live fuel load ($t\ ha^{-1}$)		SAV ($m^2\ m^{-3}$)			δ (cm)	M_x (%)	Heat ($kJ\ kg^{-1}$)		σ (m^{-1})	ρ_b ($kg\ m^{-3}$)	β/β_o
		1 h	10 h	100 h	Herb	Woody	1 h	Herb	Woody			Dead	Live			
GA-litter	Static	5.78	1.11	3.76	–	–	5258	–	–	22	31	18622	–	4889	4.84	1.12
Pub-litter[A]	Static	7.93	0.99	3.06	–	–	9186	–	–	19	30	18622	–	9106	6.30	2.43
GA-grass	Static	0.52	–	–	–	–	12781	–	–	22	14	18622	–	12781	0.24	0.12
Pub-grass[B]	Static	2.80	–	–	–	–	9842	–	–	31	15	18622	–	9842	0.90	0.37
GA-shrub	Static	3.72	0.67	0.15	3.61	2.83	6307	5353	4805	153	25	21848	18379	5595	0.72	0.19
Pub-shrub[C]	Static	4.00	0.95	0.12	5.00	2.24	6710	5490	4570	140	34	20000	20000	5822	0.88	0.23
GA-heath	Static	3.82	0	0	0	8.92	8921	–	10526	65	38	19533	21739	10098	1.96	0.82
Stand-GS3[D]	Dynamic	0.67	0.56	0	3.25	2.80	5906	5249	5249	55	40	18622	18622	5294	1.33	0.33
Cust-1[E]	Static	5.25	0	0	0	5.36	7385	–	10560	41	40	18912	22810	9269	2.59	1.01
FCCS-1[F]	NA	1.18	0	0	4.60	4.90	5904	4920	5904	38	25	18622	18622	5522	2.81	0.72

Abbreviations of fuel model parameters follow Table 1. Fuel model characteristics: σ: characteristic SAV; ρ_b: bulk density; β/β_o: relative packing ratio.

Published fuel model parameters are reported in: [A] Grabner *et al*. 1997; [B] Sneeuwjagt 1976; [C] van Wilgen 1984, with subsequent variation of the fuel bed depth from 91 cm to 140 cm in van Wilgen *et al*. (1985). [D] Standard fuel model GS3 from Scott and Burgan (2005). [E] Fuel model for heathland fuels customized using average values of fuel parameters from all heath stands pooled. [F] FCCS fuel input for heathland fuels customized using median values of fuel parameters from all heath stands pooled.

Table 3. Goodness-of-fit metrics of Rothermel model predictions obtained using GA-optimized and published fuel models for litter, grass and shrub studies. Probability values follow a t-test between published and GA optimized fuel model residuals means.

Fuel model	RMSE ($m\ min^{-1}$)	MAE ($m\ min^{-1}$)	MAPE (%)	Percentage Within ±25% error	MBE ($m\ min^{-1}$)	*p*-Value
GA-litter	3.02	2.02	54	22	0.05	0.045*
Pub-litter	4.95	3.91	128	11	3.15	
GA-grass	4.32	2.91	126	19	0.55	0.029*
Pub-grass	5.35	4.09	252	16	2.73	
GA-shrub	5.45	4.34	20	79	-0.38	0.147
Pub-shrub	7.18	6.17	30	57	2.10	

RMSE: root mean square error; MAE: mean absolute error; MAPE: mean percentage error; MBE: mean bias error. All fuel models parameters are reported in Table 2.

* Probability of mean separation $p<0.05$.

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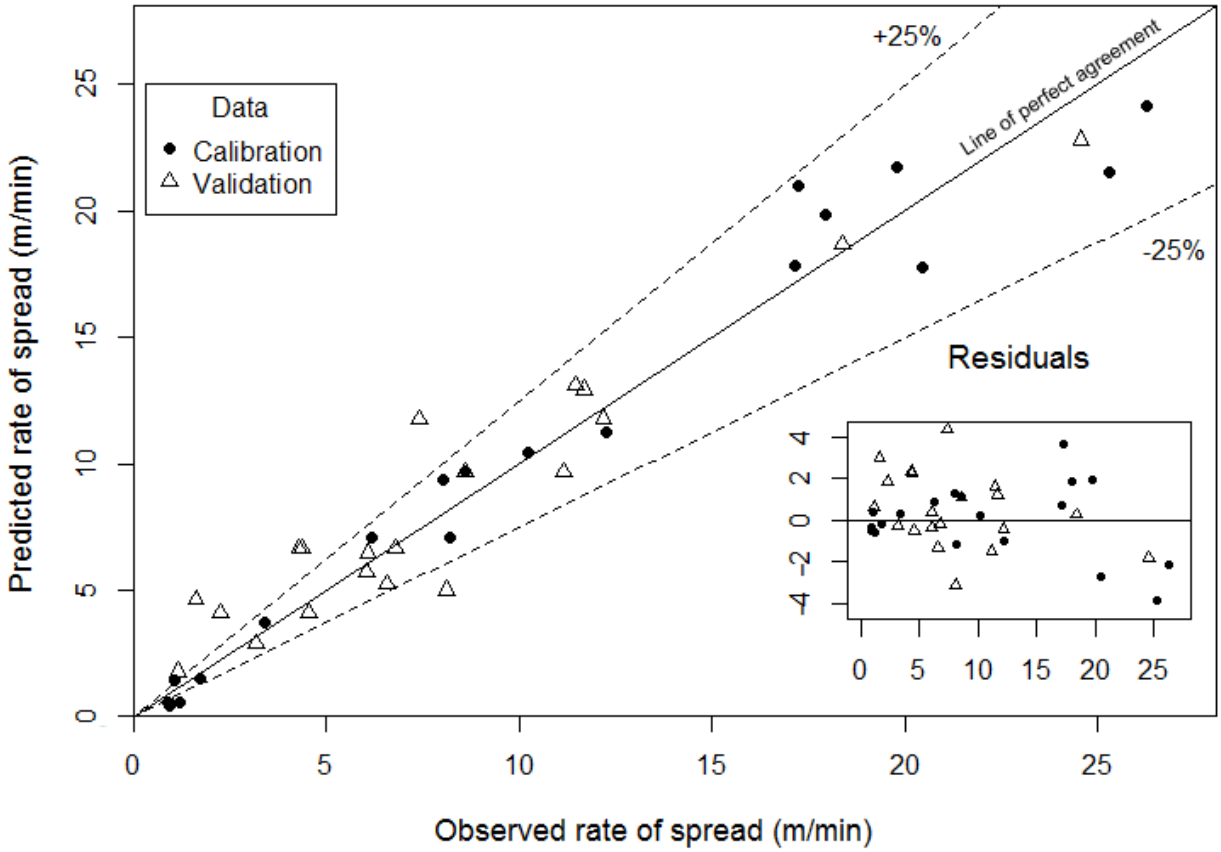
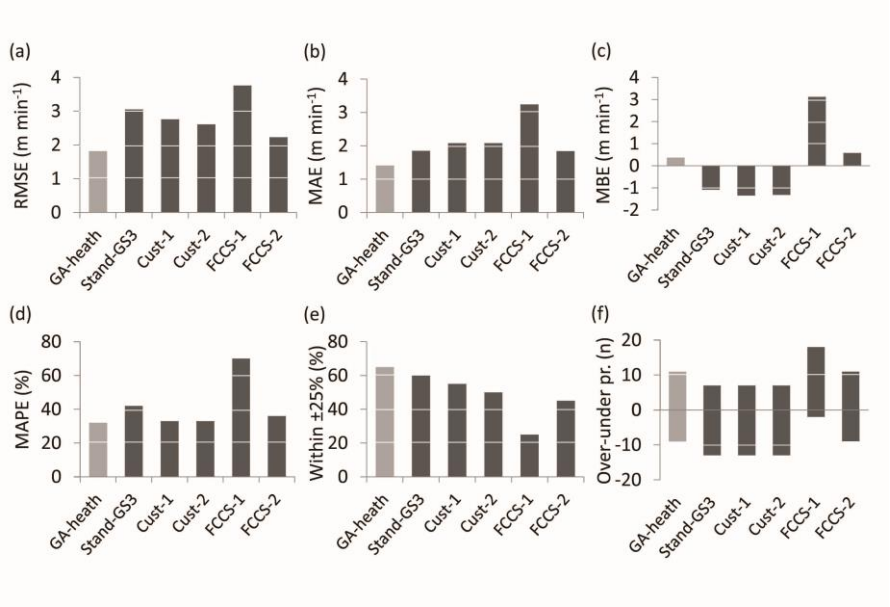
Fig. 2. Predicted vs. observed rate of spread (m min^{-1}) in the calibration (black dots) and the validation dataset (white triangles) calculated using the heathland fuel model optimized with Genetic Algorithms (GA-heath) and the *ros* ( ) function of the Rothermel Package for R. x-axis of the residual plot is the observed rate of spread, y-axis is residual (predicted – observed) rate of spread.



GA-heath had the best goodness-of-fit statistics of all the tested fuel models (Fig. 3). Among all standard fuel models within the grass-shrub group (Scott and Burgan 2005), GS3 was selected by the *bestFM* ( ) function as producing the most accurate predictions, with a RMSE of 3.05 m min^{-1} and a MBE of -1.09 m min^{-1}. The custom fuel models parameterized with average values of observed fuel characteristics produced a slightly better fit, but increased underprediction. The model for all stands pooled produced a RMSE of 2.76 m min^{-1} and a MBE of -1.35 m min^{-1}; the model using site-specific fuel data had a RMSE of 2.61 m min^{-1} and a MBE of -1.32 m min^{-1}. Both custom fuel models displayed a higher bulk density in comparison to GA-heath, as well as a higher relative packing ratio (Table 2), which was 1.01 for the model customized by average values from the overall fuel inventory, and ranged from 0.69 to 1.11 when using average values from each fire site.

An interesting result was obtained when simulating ROS using the Rothermel model reformulation implemented in FCCS. When using median values from the overall fuel inventory, FCCS overpredicted ROS and produced the worst fit statistics (Fig. 3) with a RMSE of 3.28 m min^{-1} and a MBE of 2.58 m min^{-1}. However, when using median values at the plot scale, FCCS performance improved and approached the standard fuel model GS3 with a RMSE of 2.87 m min^{-1} and a MBE of 1.55 m min^{-1} (Fig. 3).

Fig. 3. Statistics of heathland fuel models performance against the validation dataset: (a) Root mean square error (m min^{-1}); (b) Mean absolute error (m min^{-1}); (c) Mean bias error (m min^{-1}); (d) Mean absolute percent error (%); (e) Percentage within ±25% error; (f) Number of over and under predictions. GA-heath: heathland fuel model calibrated by Genetic Algorithms; Stand-GS3:standard fuel model GS3; Cust-1: custom fuel model for *Calluna* heaths, parameterized with average values from the overall fuel inventory; Cust-2: custom fuel model parameterized with site-specific average values at each fire site; FCCS-1: FCCS parameterized with median values from the overall fuel inventory; FCCS-2: FCCS parameterized with median values from each fire site.



ANOVA results showed significant differences among means of model residuals ($F_{[5,119]}$=9.91, P<0.001). The LSD test evidenced no differences between models with negative residuals (i.e., GS3 and custom fuel models parameterized with average fuel characteristics), and between models with positive residuals (FCCS models). GA-heath produced significantly different prediction from both groups, and intermediate ROS values. The regression based equivalence test confirmed trends in fit statistics (Table 4). GA-heath had the highest proportion of bootstrap sample estimates falling within the intervals of equivalence for both the slope and the intercept. Moreover, GA-heath was the only fuel model where the null hypothesis of dissimilarity was rejected for both the intercept and the slope (Table 4).

Table 4. Summary of regression-based equivalence tests for the validation of alternative fuel models against rate of spread observations. Fuel model names follow Figure 3.

Fuel Model	n	$\beta_0 \subset I_0$	$\beta_1 \subset I_1$	$C_{\beta 0}^-$	$C_{\beta 0}^+$	$I_{\beta 0}^-$	$I_{\beta 0}^+$	$C_{\beta 1}^-$	$C_{\beta 1}^+$	$I_{\beta 1}^-$	$I_{\beta 1}^+$
GA-heath	20	1.000	0.986	7.19*	8.78*	6.30	10.49	0.79*	1.15*	0.75	1.25
Stand-GS3	20	0.931	0.089	6.97	8.89	5.20	8.67	1.10	1.89	0.75	1.25
Cust-1	20	0.808	0.207	7.09	8.82	5.01	8.34	1.00	1.58	0.75	1.25
Cust-2	20	0.826	0.281	7.09	8.80	5.03	8.38	0.97	1.50	0.75	1.25
FCCS-1	20	0.497	0.933	6.99	8.87	7.95	13.26	0.68	1.16	0.75	1.25
FCCS-2	20	0.941	0.803	6.91	8.98	7.13	11.88	0.57	1.02	0.75	1.25

n, sample size; $\beta_0 \subset I_0$ and $\beta_1 \subset I_1$,proportion of bootstrap sample estimates that fall into their intervals of equivalence of the intercept ($I_{\beta 0}^-$, $I_{\beta 0}^+$) and the slope ($I_{\beta 1}^-$, $I_{\beta 1}^+$), respectively. The joint two one-sided 95% confidence intervals for the intercept and slope are ($C_{\beta 0}^-$, $C_{\beta 0}^+$) and ($C_{\beta 1}^-$, $C_{\beta 1}^+$), respectively. The former should fall within the intercept interval of equivalence ($I_{\beta 0}^-$, $I_{\beta 0}^+$) = I ± 25%, and the latter within the slope interval of equivalence ($I_{\beta 1}^-$, $I_{\beta 1}^+$) = 1 ± 0.25.

*Values for which the null hypothesis of dissimilarity has been rejected, at p <0.05.

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Discussion

The first objective of this research was to test if fuel model optimization by Genetic Algorithms improves the accuracy of the Rothermel fire spread model. We showed that GA are effective algorithm for building and calibrating custom fuel models. When tested on published data of fuels and ROS recorded in litter, grass and shrub fuels, predictions of GA-optimized fuel models had the highest goodness-of-fit, least prediction bias and lower model error. Only in the grass case, MAPE remained higher than 75%, i.e., the worst class of model performance in Cruz and Alexander (2013). However, grass fuels in Sneeuwjagt (1974) and Sneeuwjagt and Frandsen (1977) were highly variable in 1-h load and SAV (Table 1), and could hardly be described by a unique fuel model. This is probably the reason why, in their second study, Sneeuwjagt and Frandsen (1977) built separate fuel models for each of their fire experiments.

Compared to previous approaches to build fuel models (Hough and Albini 1978; Cruz and Fernandes 2008), GA optimization has the advantage of exploring a continuous rather than discrete search space for each fuel model parameter, being computationally effective thanks to stochastic rather than deterministic search rules, and being reproducible in terms of both algorithm setting and its implementation. Moreover, it does not require site-specific inventories: ranges for fuel model parameters can be set after literature data, which makes the parameterization process easier and more cost-effective.

The purpose of calibrating a fuel model is to provide the best combination of model parameters that maximizes the fit between observed and predicted fire behaviour, and not to exactly reproduce fuel characteristics observed in nature (Burgan 1987; Scott and Burgan 2005; Cruz and Fernandes 2008). Following previous studies (Hough and Albini 1978; Burgan and Rothermel 1984; Cruz and Fernandes 2008), we assumed that fuel model parameters are not necessarily set to mean or median values measured in the field, as required by other fire behaviour modelling systems (e.g., Sandberg *et al.* 2007). In fact, fire is not driven by average fuel conditions because it follows the path of least resistance, i.e., the combustion wave spreads on the finest and driest of the fine fuels (Rothermel 1972; Cheney 1990; Cruz and Fernandes 2008). On the other hand, in GA the solution depends on the initial population which is randomly generated from a defined search space. A wider search space leads to a better fitness (Chen *et al*. 2012), but care must be taken that the optimal parameter set is physically meaningful (Ohenoja and Leiviskä 2010). In fact, highly stylized fuel models that differ from reality not only for the raw parameter values but also for their ratios, e.g., the proportion of live to dead fuels, or that include fuel parameters not observed in the field, have been criticized (Sandberg *et al.* 2007). For these reasons, we decided to constrain the search space of each fuel model parameter to their minimum and maximum values reported in the literature.

The second aim of our research was to apply GA optimization to dry heathlands characterized by a mixed grass-shrub fuel complex. Studies that validate fire spread models almost always incur in a fundamental scale mismatch. The Rothermel spread model is a 1-dimensional model that simulates the spread of a fire as it burns by a point, whereas fire behaviour observations are usually taken at a stand or unit level, thereby representing the average ROS across the sequence of instantaneous flame front velocities. Our study reduced the difference between scales by measuring ROS at the microplot scale, rather than averaging the rate of spread from the ignition to the endpoint of the fire experiment.

The Rothermel model initialized with GA-heath produced a satisfactory goodness-of-fit against the validation dataset. In particular, MAPE was 32%, i.e., lower than what is considered an acceptable error for ROS simulation (Cruz and Alexander 2013). GA-heath produced a better goodness-of-fit than both the GS3 standard fuel model, and custom fuel models built using mean values from the field inventory. Interestingly, the GA-heath model slightly overestimated ROS, while both standard and custom fuel models were affected by the characteristic underestimation bias attributed to the Rothermel model (Cruz and Alexander 2013).

The original Rothermel model was reported to produce better predictions when fuel models are customized with site-specific fuel values (Sneeuwjagt and Frandsen 1977; Davies 2006). However, when comparing custom fuel models built from site-specific fuel parameters rather than means for all stands pooled, we found only slight improvements. On the contrary, the performance of FCCS improved markedly when using site-specific fuel models. This supported the intent of the Rothermel model reformulation implemented in the FCCS modelling system, which is to simulate surface fire behaviour for any wildland fuelbed using site-specific properties (Sandberg *et al.* 2007). However, the accuracy of FCCS was comparable to the standard fuel model GS3, and did not match that of GA-heath. Finally, GA-heath was the only model where equivalence testing of predicted vs. observed ROS was successful for both the intercept and the slope (i.e., the null hypothesis of dissimilarity from 0 and 1, respectively, was rejected).

We constrained the search space for GA optimization using fuel characteristics (e.g., SAV, moisture of extinction, heat content) for *Calluna* heathlands throughout their range of distribution (Fernandes and Rego 1998; Spielmann 2009; Davies and Legg 2011; Santana and Marrs 2014). However, GA-heath differed substantially from a fuel model built by Davies (2006) for Atlantic heathland dominated by *Calluna*. The main difference between the two fuel models regards the relative packing ratio, which was 1.46 in Davies (2006) and 0.82 in GA-heath. Besides the methods used to build the custom fuel model, we believe this result is due to a difference in the experimental conditions under which ROS datasets were built. In Davies (2006), experiments were carried out under faster winds up to 27 km h^{-1} at midflame height, with a mean of 11 km h^{-1} (Davies *et al.* 2009), as opposed to less than 8 km h^{-1} in this study. Conversely, measured ROS was higher in the present study (mean: 10.4 m min^{-1}; range: 0.9-26.3 m min^{-1}) in comparison to Davies *et al*. (2009) observations (mean: 4.4 m min^{-1}; range: 0.5-12.6 m min^{-1}). These conditions led GA optimization to calibrate a fuel model with a lower relative packing ratio than the one in Davies (2006) model, i.e., more sensitive to wind speed (Rothermel 1972).

This raises the question about the applicability of fuel models calibrated using site-specific parameters outside their domain of calibration. Rather than going through the effort of calibration to obtain a fuel model valid only locally, some authors have modelled fire behaviour by regression against the set of locally observed fuel and site parameters (e.g., Fernandes *et al*. 2001; Davies *et al*. 2009). However, using Rothermel's set of equation is preferable for a number of reasons: (1) as a semi-empirical model, Rothermel's has a higher realism and generality than empirical models, which are only valid for the fuel complexes on which they are calibrated; (2) Rothermel's model is run with a standard set of physical descriptors of the fuel complex, as opposed to empirical regressions that are fitted against as few fuel descriptors as possible. Consequently, as opposed to Rothermel model, regression fuel parameters may change from case to case, making it difficult to use these models within fire prediction systems. To reach a compromise between generality and accuracy of a fuel model, one should design the calibration dataset by including the widest range of experimental conditions.

In this study, optimization focused on fitting fuel models against ROS estimates. On the other hand, in wildland and prescribed burning planning, analysts must select a fuel model that provides the most accurate estimates for both ROS and flame length (van Wilgen *et al*. 1985; Alexander and Cruz 2012; Vacchiano *et al*. 2014). A possible shortcoming of this approach is that the most accurate fuel model for both ROS and flame length may not be the best for either one individually. This raises the question whether the fitness function for GA optimization should be computed against ROS, flame length, or both simultaneously. In our opinion, calibration against ROS should be prioritized, because: (i) flame length measurements are usually less precise than ROS (Alexander and Cruz 2012), and therefore are responsible for a higher uncertainty in fire behaviour simulation (Cruz and Alexander 2013); (ii) for surface fire, flame length is usually computed from fireline intensity according to the empirical relationship of Byram (1959). In turn, fireline intensity is a function of ROS and reaction intensity computed by the Rothermel model (Rothermel 1972), and flame front residence time estimated

according to Anderson (1969). This would introduce a recursive structure in the model to be optimized; (iii) the parameters of Byram's equation may be inappropriate in several fuel conditions (Alexander and Cruz 2012) and thus lead to unrealistic fuel models. However, Byram's equation parameters can be customized (Cheney 1990; Alexander and Crux 2012). A promising approach could be first to fit a fuel model by GA optimization against ROS measurements, and then use it to model flame length by optimizing the parameters of Byram's equation against observations. The result is a fuel model that provides accurate ROS simulations associated to a custom flame length-fireline intensity relationship.

Conclusion

GA optimization proved a useful and practical tool to calibrate fuel models and improve Rothermel model predictions. The fuel model calibrated by GA optimization for heathland vegetation produced accurate fire behaviour predictions and can be used to simulate surface fire rate of spread for winter prescribed burning planning in *Calluna* stands in the building phase. However, it is not appropriate for pure *Calluna* stands, typical of Atlantic European heathlands (Davies *et al.* 2009); rather, it applies to mixed *Calluna* and grass stands typical of dry heathlands of both Central and Southern Europe. Moreover, it does not apply to very dry conditions (dead fuel moisture <10%) and wind speed at midflame height >10 km h^{-1}.

Compared to previous fuel model optimization methods (e.g., Hough and Albini 1978; Cruz and Fernandes 2008) GA explores a continuous rather than a discrete space of fuel parameters, is reproducible, and is more computationally effective because it uses stochastic rather than deterministic search rules. Moreover, GA optimization has the advantage of not requiring field sampling of fuel characteristics, as long as the search space for fuel model parameters can be inferred from the existing literature.

We propose GA optimization as a viable method to calibrate custom fuel models in fire modelling systems based on the Rothermel fuel model concept, which simulate ROS both at the stand scale (e.g., Reinhardt and Crookston 2003; Andrews 2013), and at the landscape scale (Ferragut *et al.* 2008; Finney *et al.* 2011). However, additional research is needed to test how alternative settings of GA rules (e.g., population size, mutation rate) may influence the calibration of fuel models in both reducing premature convergence to non-optimal solutions, and computational efficiency. To test GA optimization, we designed the *gaRoth* ( ) function in the Rothermel Package for R (Vacchiano and Ascoli 2014), which requires a minimum of two ROS observations and searches the set of fuel model parameters that minimizes root mean square error of forward fire rate of spread simulated by Rothermel (1972) model against observed data.

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